

context. Although the algebraic complexity is increased considerably over that of the infinitely wide plate problem, no essential difficulty arises in the computation. However, with regard to the clamped edge plate, the natural modes are not the most advantageous to use. (The natural modes of a clamped plate are not available in analytical form, of course. Here reference is made to the clamped beam modes.)

As one of the writer's students has discovered, this is because of the necessity of solving an inhomogeneous equation for the stress function F , where the plate deflection W is (temporarily) considered known. In obvious notation (this is one of Von Karman's large deflection equations),

$$\nabla^4 F / Eh = (\partial^2 W / \partial x \partial y)^2 - (\partial^2 W / \partial x^2)(\partial^2 W / \partial y^2) \quad (1)$$

In the usual Galerkin approach, W is expanded as

$$W = \sum \sum a_{mn}(t) \phi_m(x) \phi_n(y) \quad (2)$$

For the natural modes of a clamped beam,

$$\phi_m(x) = \cosh G_m x - \cos G_m x + \alpha_m (\sin G_m x - \sinh G_m x) \quad (3)$$

where G_m and α_m are characteristic constants. This choice of ϕ_m leads to considerable algebraic complexity in solving for F from (1) and performing subsequent integrations. A better alternative is

$$\phi_m(x) = \cos(m-1)\pi x - \cos(m+1)\pi x \quad (4)$$

This expansion has been used previously for linear plate problems⁶ and has proven quite satisfactory for the non-linear problem as well.

References

- ¹ Humphreys, J. S., "On dynamic snap buckling of shallow arches," AIAA J. **4**, 878-886 (1966).
- ² Gallagher, A. P. and Mercer, A. M., "On the behavior of small disturbances in planar Couette flow," J. Fluid Mech. **13**, Pt. 1, 91-100 (May 1962).
- ³ Ketter, D. J., "Coupled panel cavity vibrations," AIAA J. **3**, 2164-2166 (1965).
- ⁴ Ketter, D. J., "Addendum: 'Coupled cavity panel vibrations,'" AIAA J. **4**, 768 (1966).
- ⁵ Dowell, E. H., "Nonlinear oscillations of a fluttering plate," AIAA Preprint 66-79 (1966); also AIAA J. **4**, 1267-1276 (1966).
- ⁶ Dowell, E. H. and Voss, H. M., "The effect of a cavity on panel vibration," AIAA J. **1**, 476-477 (1963).

Reply by Author to E. H. Dowell

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THE author wishes to thank E. H. Dowell for his useful comments on Ref. 1 regarding clamped beam and panel modes. Since the "bootstrap" technique referred to by Dowell² for evaluation of coefficients such as

$$\int_0^\pi \phi_s \phi_n'' d\xi$$

is not explained clearly in his Ref. 3 and appears to be such a powerful shortcut, perhaps a few additional details would be of value. Essentially, the evaluation of integrals such as the preceding can be accomplished by successive partial integration, making use of the cyclic properties of the beam modes in that they satisfy the beam equation $\phi_n'''' = m_n^4 \phi_n$. If sufficient partial integrations are performed, using the beam equation at different times for both ϕ_s and ϕ_n , the original integral is regained eventually, plus several terms

to be evaluated at the integration limits. The latter terms are much easier to evaluate than the process performed by the author in Ref. 1 of forcing the integration through via a series expression.

One further point not mentioned in Dowell's Refs. 2 or 3 is that for any mode index higher than unity the multiplicative factor

$$\alpha_n = \frac{\cos m_n \pi - \cosh m_n \pi}{\sin m_n \pi - \sinh m_n \pi}$$

(in Ref. 1 notation) that appears in the mode shapes is 1.00 to at least three significant figures. This further simplifies the expressions for the integral coefficients for almost all the terms needed. Such approximate expressions as derived in Ref. 1 (albeit by a tedious route) are identical with comparable expressions derived from Dowell's references.

References

- ¹ Humphreys, J. S., "On dynamic snap buckling of shallow arches," AIAA J. **4**, 878-886 (1966).
- ² Dowell, E. H., "Comment on 'On dynamic snap buckling of shallow arches,'" AIAA J. **4**, 1887-1888 (1966).

Errata: "Electrical Conductivity of Partially Ionized Gases"

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[AIAA J. **4**, 1012-1019 (1966)]

THE following correction should be made to the above article.

- 1) Page 1014, fourth line in Eq. (12) should read:

$$q^{02} = 4 \sum_{j=3}^{\nu} n_j \left(\frac{35}{4} \Omega_{1j}^{11} \right) - \dots$$

- 2) Page 1015, second column, line 6 should read:

$$\epsilon_m = \frac{1}{4}(m^2 - m)\gamma_m$$

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Erratum: "Relationship between Filler Distribution and Uniaxial Rupture of Composite Solid Propellants"

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EQUATION (13) should read

$$\lambda_r = [(\lambda/J_s) - V_f]/(1 - V_f)$$

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